Advanced Automation and Control

September 17, 2019

Surname ____________________ Name ____________________

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{ll} \max_{x_1,\delta_1,\delta_2} & -x_1 + \delta_1 - \delta_2 \\ & \delta_1 + x_1 \leq 0 \\ & \delta_2 + 1.5x_1 \geq 0 \\ & x_1 \leq 0 \\ & \delta_1, \delta_2 \in \{0, 1\} \end{array}$$

2. Mr. Lemon is studying Medicine at University of Pavia. During the exam session of September he would like to give three exams: Statistics (6 Credits), Microbiology (9 Credits) and Human Genetics (5 Credits) which we will indicate respectively as E1, E2 and E3. The exams are scheduled as follows: E1 - 06/09/2018, E2, 12/09/2018, E3, 18/09/2018. Assume Mr. Lemon starts studying the 1st of September and dedicates 9 hours a day to the study, except for the days he gives an exam. **If**, for example, he gives E2, then he will study 0 hours that day. Let us divide the 17 days (01/09 to 17/09, 153 hours in total) into three parts: (P1: 01-05, P2: 06-11, P3: 12-17). During P1, Mr. Lemon could divide his studying time between E1, E2, E3. During P2, between E2 and E3, etc. In order to pass an exam, he has to study at least 43 hours for E1, 50 for E2, 50 for E3. The objective of Mr. Lemon is to minimize the number of studying hours that guarantees him to pass at least two exams and collect at least 14 Credits. Is it possible to pass all the three exams?

   Please formulate the problem above as a MILP to support the decision-making of Mr. Lemon.

3. Consider the following optimisation problem

$$\begin{array}{ll} \max_{x} & -f(x) \\ & \log(x) \leq 0 \\ & x \geq 0 \end{array}$$

   where

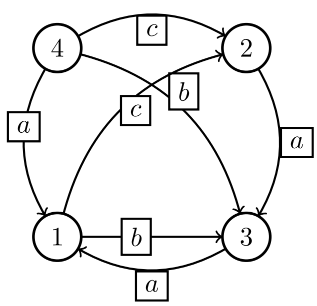
$$f(x) = \begin{cases} x^2 & x \leq 0.5 \\ x - 0.25 & x \geq 0.5 \end{cases} \tag{1}$$

   3.1 Indicate if the cost function is convex (motivate the answer).

   3.2 Depict the feasibility domain of the problem.

   3.3 Indicate if the optimisation problem is convex (motivate the answer).

4. Consider the automaton in the figure ($C = \{a, b, c\}$ is the set of control values and $S = \{1, 2, 3, 4\}$ is the set of state values) with the intermediate cost $g(x, u)$ and the terminal cost $g_2(x)$ given below



$g(x,u)$	a	b	c
1	-	1	2
2	3	-	-
3	4	-	-
4	-1	-2	-3

$$g_2(x) = \begin{cases} -1 & \text{if } x = 1 \\ 1 & \text{if } x = 2 \\ -1 & \text{if } x = 3 \\ 1 & \text{if } x = 4 \end{cases}$$

2.1 Solve the optimal control problem

$$J(x_0) = \min_{u_0, u_1} g_2(x_2) + \sum_{k=0}^{1} g(x_k, u_k)$$

using dynamic programming.

2.2 Compute the optimal control sequence for $x_0 = 4$ and for $x_0 = 3$ and the corresponding optimal cost values.